

Hawking radiation from $z=3$ and $z=1$ -Lifshitz black holes

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(Dated: November 13, 2014)

Abstract

The Hawking radiation considered as a tunneling process, by using a Hamilton-Jacobi prescription, is discussed for both $z = 3$ and $z = 1$ -Lifshitz black holes. We have found that the tunneling rate (which is not thermal but related to the change of entropy) for the $z = 3$ -Lifshitz black hole (which does not satisfy the Area/4-law) does not yield (give us) the expected tunneling rate: $\Gamma \sim \exp(\Delta S)$, where ΔS is the change of black hole entropy, if we compare with the $z = 1$ -Lifshitz black hole (BTZ black hole, which satisfies the Area/4-law).

PACS numbers: 98.80.Cq

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I. INTRODUCTION

The second law of thermodynamics was initially put forth for a system including black holes by Bekenstein [1]. It states that sum of one quarter of the area black hole's event horizon plus the entropy of ordinary matter outside never decrease with time all process. The thermodynamics properties of these objects are based on its elusive quantum description (as everything seems to indicate). Specifically, the problem of emission of particles and its relationship to the second law of thermodynamics, i. e., the entropy. Black holes can emit particles from the event horizon known as Hawking radiation [2]. Hence, the classical notion is no longer that a black hole is not as black, or in other words, the black hole radiation appears as a quantum process (as everything seems to indicate). Whilst the nature of Hawking radiation is not yet fully understood [3], this fact has established an important antecedent when exploring the evolution of a black hole and its thermodynamic behaviour taking into account the underlying holographic idea about the crucial relation between entropy and area in a gravitational system [4]. In this work we study the Hawking radiation from a couple of Lifshitz black holes ($z = 3$ and $z = 1$ cases) by following an idea proposed in [5]: the Hawking radiation from a black hole described as a tunneling process. The results for both cases are compared and discussed. The present paper is organized as follows. In Section 2, we review the $z = 3$ -Lifshitz black hole solution and its thermodynamics. In Section 3, we compute the tunneling rate (which is not thermal) for this black hole by following a prescription based on the relativistic Hamilton-Jacobi equation and we compare the obtained result with the one corresponding to $z = 1$ -Lifshitz black hole (BTZ black hole). In Section 4 we discuss our results. In this paper, the units $G=\hbar=c=1$ are used.

II. $z = 3$ -LIFSHITZ BLACK HOLE

The black hole solution is described by the following metric [6]

$$ds^2 = - \left(\frac{r^2}{l^2} \right)^3 (1 - r_+^2/r^2) dt^2 + \frac{l^2}{r^2} (1 - r_+^2/r^2)^{-1} dr^2 + r^2 d\varphi^2, \quad (1)$$

where the coordinates are defined by $-\infty \leq t \leq \infty$, $r \geq 0$ and $0 \leq \varphi \leq 2\pi$, and $r_+ = l\sqrt{M}$ is the black hole event horizon. Here M is an integration constant related to the black hole mass and l is the curvature radius of the Lifshitz spacetimes related to the cosmological

constant through $\Lambda = -13/2l^2$. The thermodynamical quantities of this black hole [7], i.e. temperature, entropy and mass are, respectively,

$$T = \frac{r_+^3}{2\pi l^4} \quad , \quad S = 2\pi r_+ \quad , \quad \hat{M} = \frac{r_+^4}{4l^4} = \left(\frac{M}{2}\right)^2, \quad (2)$$

such that $TdS = d\hat{M}$; meaning that the first law is satisfied but not the Area/4-law for the entropy: $S = 2\pi r_+ \neq \pi r_+/2 = \text{Area}/4$. This point will be very important at time to calculate the tunneling rate for this black hole and the subsequent comparison with the $z = 1$ -Lifshitz black hole (BTZ black hole). For $z \neq 1, 3$ it is not clear yet how to compute conserved quantities for asymptotic Lifshitz black holes and this fact make difficulties if we want to have a consistent thermodynamic: the thermodynamical laws should be verified for this type of black holes. Whether or not the Area/4-law is a universal law which must satisfy all black holes, independent of the dimensionality of the spacetime in which they live, is still an open question. We conclude this Section with an example (there are others) which does not appear to be of any help in finding an answer to the previous question: in a scattering process of scalar fields over a $z = 3$ -Lifshitz black hole [8], the absorption cross-section is proportional to the entropy independent whether or not the Area/4-law is fulfilled.

III. THE RELATIVISTIC HAMILTON-JACOBI EQUATION

We use a semi-classical method proposed in Ref. [9] in order to conceive the Hawking radiation as a tunneling effect. The idea is to consider a scalar particle moving in the background of the black hole where the particle self-gravitation is neglected. So, the classical action I of the particle satisfies the relativistic Hamilton-Jacobi equation

$$g^{\mu\nu} \left(\frac{\partial I}{\partial x^\mu} \right) \left(\frac{\partial I}{\partial x^\nu} \right) + m^2 = 0, \quad (3)$$

where m and $g^{\mu\nu}$ are the mass of the particle and the inverse metric tensor derived from (1) and $x^\mu = (t, r, \varphi)$. Near the horizon

$$\left(\frac{r^4}{l^6} \Delta(r) \right)_{r \rightarrow r_+} \longrightarrow \frac{2r_+^5}{l^6} (r - r_+) \quad , \quad \left(\frac{l^2}{\Delta(r)} \right)_{r \rightarrow r_+} \longrightarrow \frac{l^2}{2r_+ (r - r_+)}, \quad (4)$$

such that in this case, where the non-null inverse metric elements are

$$g^{tt}(r) = -\frac{l^6}{2r_+^5} \frac{1}{(r - r_+)} \quad , \quad g^{rr}(r) = \frac{2r_+}{l^2} (r - r_+) \quad , \quad g^{\varphi\varphi}(r) = \frac{1}{r_+^2}. \quad (5)$$

By replacing (5) in (3), and because there are two Killing vectors in the present $z = 3$ -black hole, we do $I = -\omega t + R(r) + j\varphi$, where ω and j are the energy and angular momentum of the particle respectively and $R(r)$ is the geometric content of the spacetime under consideration, we have the following integral expression for $R(r)$

$$R(r) = \pm \frac{l^4 \omega}{2r_+^3} \int \frac{dr}{r - r_+} \sqrt{1 - 2 \left(\frac{r_+^2}{\omega l^3} \right)^2 r_+ \left(m^2 + \frac{j^2}{r_+^2} \right) (r - r_+)}, \quad (6)$$

expression from which we rescue the imaginary part (classically forbidden process) which is related to the Boltzmann factor for emission at the Hawking temperature

$$R(r) = \pm \frac{l^4 \omega}{2r_+^3} (i\pi) \implies I = -\omega t \pm \frac{l^4 \omega}{2r_+^3} (i\pi) + j\varphi. \quad (7)$$

The \pm sign in (7) correspond to outgoing and ingoing particles, respectively. Given that in the classical limit all is absorbed by the black hole (with no reflection) we write ImI corresponding to outgoing particles

$$ImI = \frac{l^4 \omega \pi}{2r_+^3} = \frac{\omega}{4T} = \frac{\pi l \omega}{2^{5/2}} \hat{M}^{-3/4}, \quad (8)$$

where $r_+^2/l^2 = 2\sqrt{\hat{M}}$ ($\Re I$ corresponds to ingoing particles, i. e., particles falling through the horizon of the black hole). We note that $2ImI = \omega T/2$ and already this fact shows the non-thermal nature of the emission; for thermal emission $2ImI = \omega T$. Now, when a particle with energy ω tunnels out, the mass of the black hole changed into $\hat{M} \rightarrow \hat{M} - \omega$ ($l\sqrt{\hat{M}} \rightarrow l\sqrt{\hat{M} - \omega}$, where $l\sqrt{\hat{M}}$ is the horizon before pair-creation and $l\sqrt{\hat{M} - \omega}$ the horizon after pair-creation). Therefore we put

$$ImI \rightarrow -\frac{\pi l}{2^{5/2}} \int_{\hat{M}}^{\hat{M}-\omega} d(\hat{M} - \omega) (\hat{M} - \omega)^{-3/4}, \quad (9)$$

and we obtain

$$ImI = -\frac{1}{4} \left[\left(1 - \frac{\omega}{\hat{M}} \right)^{1/4} - 1 \right] S = -\frac{1}{4} \Delta S, \quad (10)$$

where $S = 2\pi r_+$ is the $z = 3$ -Lifshitz black hole entropy. In the WKB approximation the tunneling rate (tunneling probability for the classically forbidden trajectory from inside to outside the horizon) is given by $\Gamma \sim \exp(-2ImI)$ and for this black hole we find Therefore, the tunneling rate $\Gamma \sim \exp(-2ImI)$ for this black hole is given by

$$\Gamma \sim \exp\left(\frac{1}{2}\Delta S\right), \quad (11)$$

and we note that (11) differs from the result $\Gamma \sim \exp(\Delta S)$ which is valid for black holes that satisfy the Area/4-law. If we repeat the scheme done before for the BTZ black hole ($z = 1$ -Lifshitz black hole) we find [10]

$$\Gamma \sim \exp(\Delta S), \quad (12)$$

where

$$\Delta S = \left[\left(1 - \frac{\omega}{M_{BTZ}} \right)^{1/2} - 1 \right] S, \quad (13)$$

being $S = \pi r_+/2 = 2\pi r_+/4 = \text{Area}/4\text{-law}$ and $r_+ = l\sqrt{M} = 2\sqrt{2}l\sqrt{M_{BTZ}}$, such that $dM_{BTZ} = TdS$ (first law) and $T = r_+/2\pi l^2$ [6].

IV. CONCLUDING REMARKS.

We have computed the tunneling rate for both $z = 3$ -Lifshitz black hole, which does not satisfy the Area/4-law and we find $\Gamma \sim \exp(\Delta S/2)$, and $z = 1$ -Lifshitz black hole (BTZ), which satisfies the Area/4-law and we found $\Gamma \sim \exp(\Delta S)$. It is not yet clear how to compute conserved quantities in asymptotic Lifshitz black holes and so, we can not visualize the scope of our results if we are thinking in a complete thermodynamical description of these type of black holes in the framework, for instance, of new massive gravity. Nevertheless, we apologize that the difference in the tunneling rates obtained can be a signal to discriminate between black holes which satisfy the Area/4-law and those who do not. Finally, in the literature we can find discussions about dependence on the type of coordinates used to describe black holes and then calculate tunneling rates (see [11] and references therein). However, we believe that the thermodynamic properties of these, if they do, must be independent of that choice, as it should be if we accept (assume?) that black holes are thermal objects.

ACKNOWLEDGMENTS

This work was supported by Fondecyt Grant No. 1110076 (SL) and VRIEA-DI-PUCV Grant No. 037.377/2014, Vice Rectoría de Investigación y Estudios Avanzados, Pontificia Universidad Católica de Valparaíso (SL). We thank U. Raff for a careful reading of the

manuscript. Also we acknowledge Dr. Joel Saavedra for helpful discussions.

- [1] J. D. Bekenstein, *Nuovo Cim. Lett* **4**, 737 (1972), *Phys. Rev. D* **7**, 2333 (1973) and *Phys. Rev. D* **9**, 3292 (1974).
- [2] S. W. Hawking, *Nature* **30**, 248 (1974), S. Hawking, *Commun. Math. Phys.* **43** (1975) 199-220.
- [3] R. Schützhold and W. G. Unruh, arXiv:1308.2159 [gr-qc].
- [4] K. Jensen and A. O'Bannon, *Phys. Rev. D* **88** (2013) 106006, A. Bhattacharyya, M. Sharma and A. Sinha, *JHEP* **1401** (2014) 021.
- [5] M. K. Parikh and F. Wilczek, *Phys. Rev. Lett.* **85** (2000) 5042-5045.
- [6] Eloy Ayón-Beato et al, *Phys. Rev. D* **80** (2009) 104029.
- [7] Yun Soo Myung and Taeyoon Moon, *Phys. Rev. D* **86** (2012) 024006.
- [8] S. Lepe et al, *Phys. Rev. D* **86** (2012) 066008.
- [9] M. Angheben et al, *JHEP* **0505** (2005) 014.
- [10] Xiaokai He and Wembiao Liu, *Phys. Lett. B* **653** (2007) 330-334.
- [11] Ren Jun, Jia Meng-Wen and Wei Yi-Huan, *Chin. Phys. B* **22** (2013) 11, 110401.